Mathematische Methoden zur Analyse von Zeitreihen komplexer Systeme

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Aufgabenblatt

Aufgabe 1 Kalman-Filter

Commander Riker left the Enterprise in a shuttle but gets into trouble in a mysterious, viscous interstellar medium. This medium effects the radial movement of the shuttle and, moreover, due to the *fluctuation-dissipation theorem*, exposes the dynamics of the shuttle to small random perturbations. To beam him back to the Enterprise Captain Picard needs the best available estimation of the radial velocity of the shuttle. Unfortunately the velocity data of the shuttle recorded by the long-range scanner are disturbed by perturbations caused by the mysterious, viscous interstellar medium. Fortunately, Commander Data remembers the variances of the random perturbations of the dynamics as well as of the observation and, even more fortunately, also the coefficient of viscosity of the mysterious interstellar medium.
Help the crew to estimate the time course of the radial velocity of the shuttle.

- Taking into account the two types of perturbations one gets

$$\begin{aligned} x(t) &= a\,x(t-1) + \epsilon(t), \quad & \epsilon(t) \sim N(0, \sigma_1^2) \\ y(t) &= x(t) + \eta(t), & \eta(t) \sim N(0, \sigma_2^2) \end{aligned}$$

  Credits to Commander Data, all parameters are known: $\sigma_1^2 = 100$, $\sigma_2^2 = 100$, $a = 0.99$ and, furthermore, $x(0) = 100$ at the time of entering the m.v.i.m.

- Generate output of the long-range scanner and apply the Kalman filter to estimate Riker's velocity.

Aufgabe 2 Beziehung $x(t|t-1)$, $x(t|t)$ und $x(t|N)$ zu $x(t)$

- Betrachte den Zeitverlauf von $K(t)$, $P(t|t)$ und $P(t|t-1)$.
  Vergleiche letztere und interpretiere den Unterschied.
- Implementiere das Glättungsfilter.
- Untersuche per Kreuzkorrelationsfunktion, Kohärenz- und Phasenspektrum die Beziehung zwischen $x(t|t-1)$, $x(t|t)$, respektive $x(t|N)$ und $x(t)$ für $N = 8192$.
- Verstehe das Ergebnis.
- Artikuliere Dein Verständnis in kurzen, klaren Sätzen.