Mon. Not. R. Astron. Soc. **303,** 297–300 (1999)

On the irregular temporal behaviour of the variable star R Scuti

M. König,[1] E. Paunzen[2] and J. Timmer[3]

[1] *Institut für Astronomie und Astrophysik, Astronomie, Universität Tübingen, Waldhäuser Str. 64, D-72076 Tübingen, Germany*
[2] *Institut für Astronomie der Universität Wien, Türkenschanzstr. 17, A-1180 Wien, Austria*
[3] *Fakultät für Physik, Albert-Ludwigs-Universität, Hermann-Herder Str. 3, D-79104 Freiburg, Germany*

Accepted 1998 October 14. Received 1998 October 12; in original form 1997 September 23

ABSTRACT

We comment on the recent results of a temporal analysis of R Scuti, a star of RV Tauri type, presented by Buchler et al. As an alternative approach, we apply the technique of linear state space analysis to describe the irregular temporal behaviour with a linear stochastic process. We find evidence that the variability of R Scuti can be explained by the superposition of two stochastically driven damped oscillators.

Key words: methods: data analysis – stars: individual: R Sct – stars: variables: other.

1 INTRODUCTION

The optical light curves of metal-poor Population II Cepheids (W Virginis and RV Tauri) often exhibit irregular pulsations (e.g. Kukarkin 1975). It can be shown that a deterministic model of a multi-periodic Fourier decomposition is not compatible with the observational data (Buchler et al. 1996). The irregularity in the pulsation pattern can be attributed to a manifestation of low-dimensional chaos (Buchler & Kovacs 1987). In a recent paper (Buchler et al. 1996), theoretical conclusions of such a non-linear model are applied to observational data on the long-term light curve of R Scuti provided by the American Association of Variable Star Observers (AAVSO). Recently, the AAVSO has completed a compilation of the observational data on some RV Tauri stars. Of these, the R Scuti light curve is particularly long (*c.* 31 yr) with a relatively good temporal sampling and a large pulsation amplitude.

The AAVSO light curves of the RV Tauri star R Scuti are available for the intervals 1963–85, 1985–90 and 1991–95. The observations include the Julian date of the observation and the visual magnitude of the variable at the reported time (and further information). In order to obtain comparable data sets, we followed the data preparation method of Buchler et al. (1996) by combining all observation sets. Then, we binned the data to 2.5-d time bins, arriving at 4714 time bins for the 1963–95 observations. Finally, we subtracted a low-order polynomial fit to reduce the influence of long-term variations with time-scales of a few years or longer (this subtraction has no influence on the results given below). Fig. 1(a) shows the resulting time series with a mean of 5.86 mag and a rms of 0.70 mag, respectively.

The time series displayed in Fig. 1(a) clearly shows a strong non-linear behaviour of the irregular pulsations. This apparent non-linearity is at the root of the failure to describe the dynamics of the light curve (Buchler et al. 1996) with linear models [i.e. auto-regressive (AR) models]. We believe that this property is caused by a fact which is apparent already when the physical units are considered that scale the light curve. The photometry signal is given in units of magnitudes. If the emission properties of a variable star are examined, the signal values to be processed should be linearly correlated with the emitted photon flux S of the star, thus

$$\mathrm{mag}_1 - \mathrm{mag}_2 = -2.5\log_{10}\left(\frac{S_1}{S_2}\right). \tag{1}$$

Following equation (1), we have transformed the AAVSO R Scuti light curve into a time series which represents the photon flux (Fig. 1b). The resulting light curve displays an enhanced symmetry about the mean value, as compared with the magnitude light curve in Fig. 1(a). Therefore, we have assumed an underlying linear stochastic process to have generated the transformed light curve of R Scuti. To handle the observational noise, we have used the technique of a linear state space model, which models the noise and thus allows an estimation of the underlying dynamics of an AR process (König & Timmer 1997).

2 TEMPORAL ANALYSIS

In this section we briefly introduce the linear state space model (LSSM). For a detailed discussion, see Honerkamp (1993). The LSSM is a generalization of the AR model proposed by Yule (1927), which inspired the analysis of the variability of Wolf's sunspot numbers.

A given discrete time series $x(t)$ is considered as a sequence of correlated random variables. The AR model expresses the temporal correlations of the time series in terms of a linear function of its past values plus a noise term, and is closely related to the stochastic differential equation which describes the system's dynamics (a detailed description can be found in Priestley 1992). The fact that $x(t)$ has a regression on its own past terms gives rise to the terminology 'autoregressive process' (for detailed discussions see Scargle 1981 and Priestley 1992). A time series is thus a realization of a stochastic process or, even more precisely, the observation of a realization of the process during a finite time interval. The 'AR process' variable $x(t)$ is recursive and expressed in linear

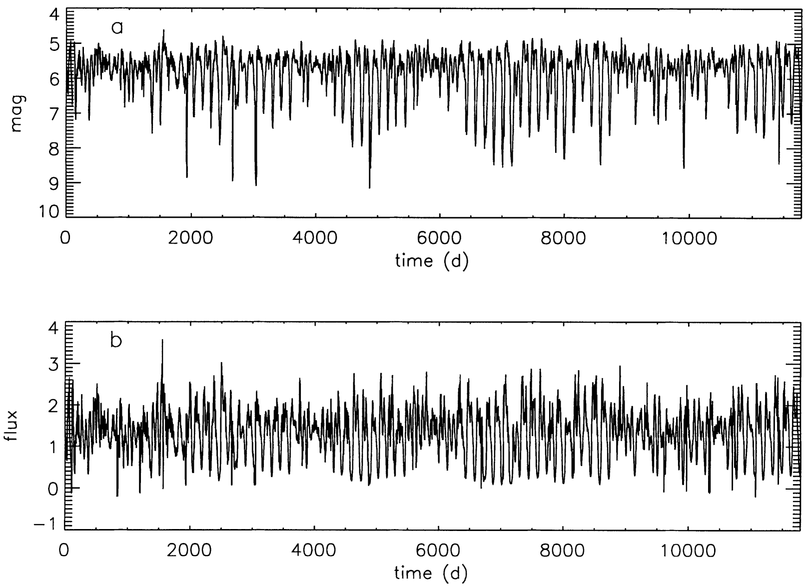

Figure 1. AAVSO R Scuti light curve 1963–95 with (a) ordinate in magnitudes and (b) ordinate in arbitrary photon flux units (zero is JD 243 8295.5).

combinations of $x(t-1), x(t-2), \ldots$ plus an uncorrelated (Gaussian) white noise process, $\epsilon(t)$ (assuming a normal distribution as a consequence of the central limit theorem).

$$x(t) = \sum_{i=1}^{p} a_i x(t-i) + \epsilon(t), \qquad \epsilon(t) \sim \mathcal{N}(0, \sigma^2). \tag{2}$$

The number of terms p used for the regression of $x(t)$ determines is the order of the AR process, which is thus annotated as an AR[p]. Depending on the order p and the values of the parameters a_i, the process is represented by damped oscillators, pure relaxators or their superpositions. The formulae to transform the dynamical parameters a_i to periods and relaxation times are given in Priestley (1992).

LSSMs generalize the AR processes by explicitly modelling observational noise. The variable $x(t)$ must be estimated indirectly as it is corrupted by observational noise $\eta(t)$. The measured observation variables $y(t)$ need not necessarily coincide with the system variables $x(t)$ that provide the best description of the system dynamics. Thus an LSSM is defined by two equations, the system or dynamical equation (3) and the observation equation (4):

$$\boldsymbol{x}(t) = \mathbf{A}\boldsymbol{x}(t-1) + \boldsymbol{\epsilon}(t), \quad \boldsymbol{\epsilon}(t) \sim \mathcal{N}(0, \mathbf{Q}), \tag{3}$$

$$y(t) = \mathbf{C}\boldsymbol{x}(t) + \eta(t), \qquad \eta(t) \sim \mathcal{N}(0, R). \tag{4}$$

This definition is a multivariate description of equation (2), i.e. the AR[p] process is given as a p-dimensional AR process of order one, with a matrix $\mathbf{A}$ that determines the dynamics. The matrix $\mathbf{C}$ maps the unobservable dynamics to the observation. The vector-valued noise variable $\boldsymbol{\epsilon}(t)$ represents the dynamical noise and is governed by a covariance matrix $\mathbf{Q}$. The one-dimensional observational noise $\eta(t)$ is described by its covariance R. The matrices $\mathbf{A}$, $\mathbf{Q}$ and the scalar R are estimated by a maximum likelihood algorithm (Honerkamp 1993).

A necessary condition that the LSSM AR[p] model fits the data is that the residuals between the measured and the estimated time series [i.e. the difference $y(t) - \mathbf{C}\boldsymbol{x}(t)$] should be undistinguishable from white noise, i.e. the time series of prediction errors should be uncorrelated. The variance of the observational noise is correlated with the variance of the residuals given in Table 1 (see König & Timmer 1997 for details). A Kolmogorov–Smirnov test was computed in order to test for a flat spectrum and to quantify the goodness of fit for the fitted LSSM AR[p] models (Table 1). This test has been used to obtain a reliable criterion for the validity of the a priori assumption of Ansatz functions in the study of non-periodic temporal behaviour in the X-ray light curves of active galactic nuclei (König, Staubert & Wilms 1998). For details on this test and other alternative methods the papers of Honerkamp (1993) and König & Timmer (1997) should be consulted.

We applied LSSMs with different orders of the AR processes. An LSSM using an AR[0] process corresponds to a pure white noise process without any temporal correlation and a flat spectrum. The

Table 1. Results of LSSM AR[p] fits.

Order p	$\sigma^2_{\rm res}$ [a]	Period [b] (d)	τ [c] (d)	$\mathrm{KS_{test}}$ [d] %
0	1	–	–	0
1	0.339	–	37.93	0
2	0.257	93.79	39.38	4.6
3	0.252	115.63	36.54	24.5
		–	16.71	
4	0.213	141.03	1450.41	78.1
		71.21	267.95	
5	0.214	142.99	1073.09	89.1
		70.92	390.06	
		–	27.85	
6	0.211	142.58	1875.90	86.7
		71.45	515.89	
		–	17.12	
		–	11.47	
7	0.210	142.84	1340.40	66.2
		70.81	850.19	
		61.81	36.81	
		–	23.94	

[a] Variance of the residuals $y(t) - \mathbf{C}\boldsymbol{x}(t)$ (the variance of the RScuti light curve is set to 1 for clarity).
[b,c] Period and relaxation time of a damped oscillator (both values given) or a relaxator (only τ given).
[d] Probability that the residuals are white noise as computed by a Kolmogorov–Smirnov test.

Kolmogorov–Smirnov test rejects this model at any level of significance (Table 1). The LSSM AR[4] model gives a good fit to the AAVSO R Scuti data as the residual variance remains nearly constant for model orders $p \geq 4$ ($\sigma^2_{\rm res} \approx 0.21$) and the residuals are consistent with white noise. The dynamical capabilities of less complex LSSM AR models ($p = 1 \ldots 3$) only give an insufficient description of the light curve variability. On the other hand, higher order LSSM AR[p] fits ($p > 4$) yield no improvement of the dynamical description of the LSSM AR[4] model, as the additional relaxators and damped oscillators are negligible. Any further increase of the model order will not reduce the variance significantly. Oscillators and relaxators which might occur in unnecessarily more complex LSSMs should be highly damped and can therefore be neglected. We therefore conclude that the LSSM AR(4) model supplies a reasonable description of the underlying dynamics of the light curve. As the light curve data are a composition of different observations with different statistical qualities, the statistical significance of the fitted models is affected and the variance of the residual time series slightly varies for model order $p > 4$.

We have also used the Durbin–Levinson algorithm to estimate the parameters of a competing simple AR model (see König & Timmer 1997 for details). As expected for time series containing observational noise, the characteristic time-scales are underestimated by fitting a simple AR process and the statistical test rejects the AR model. A test for white noise residuals fails, which means that there are still correlations present that cannot be modelled with an AR process.

Any initial correlation in the observed time series is removed with the subtraction of appropriate Ansatz functions of damped oscillators and relaxators, the parameters of which have been estimated (Buchler et al. 1996; König 1997).

3 DISCUSSION

We have found that the AAVSO light curve of the variable RV Tauri star R Scuti can be adequately modelled with an LSSM AR[4] model. The dynamical parameters of the LSSM AR[4] fit reveal two damped oscillators with periods of $\approx$141 and $\approx$71 d, and relaxation times of $\sim$1500 and $\sim$300 d, respectively. Higher order LSSM AR[p] models cannot improve these fits significantly. Therefore, models with augmented complexity (with additional relaxators and damped oscillators) fail to ameliorate the description of the light curve dynamics. In addition, the AR[4] model provides all features occurring in the R Scuti periodogram (see Fig. 2), especially the broad peaks which indicate the periods of the damped oscillators, the increase of power with decreasing frequency and the flattening of the periodogram at low frequencies. Please note that the 71-d peak does not occur as a harmonic of the 141-d oscillation, but represents an independent oscillating mode. The parameters that determine both oscillations are independently estimated. No relationship between the amplitudes and phases of the two oscillation modes can be detected.

Our dynamical interpretation follows the idea of a dissipative system with 'viscous' behaviour of the opacity in the outer stellar atmosphere.

This damped oscillating system releases energy mainly in two oscillating modes. We are thus led to conclude that a description of the irregular variability of R Scuti by means of a linear stochastic model class is an interesting and competitive alternative to the nonlinear description of multiperiodic chaotic models of stars.

ACKNOWLEDGMENTS

We are indebted to M. G. Firneis for her comments and proofreading. We thank K. Pottschmidt, B. Stelzer and J. Wilms for helpful discussions, as well as an anonymous referee for further improvements of the script. This research has made use of AAVSO archive data. EP acknowledges support of this research within the working group Asteroseismology-AMS with funding from the Fonds zur Förderung der wissenschaftlichen Forschung (project S7303-AST).

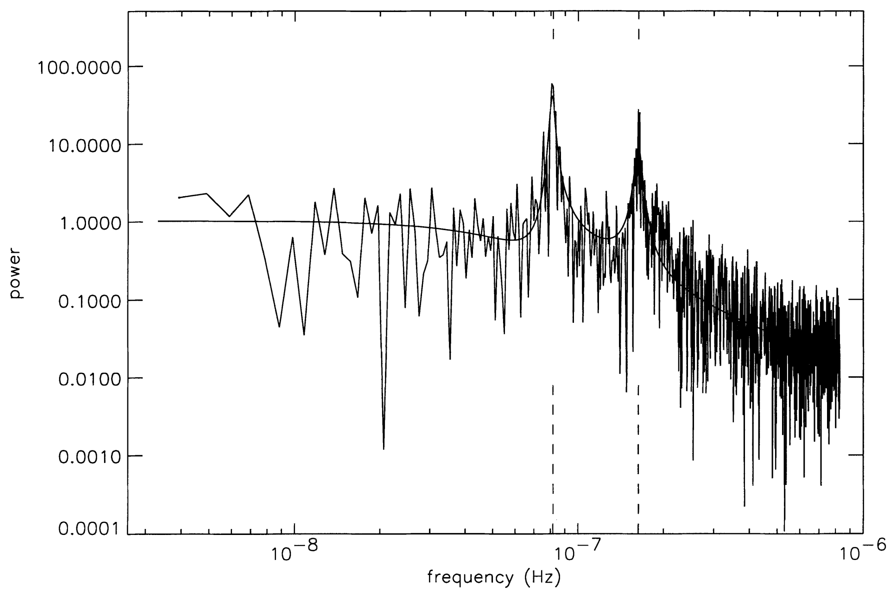

Figure 2. Periodogram of the AAVSO R Scuti photon flux time series. The dashed lines display the centres of the broad period peaks at 141 d (8.21×10^{-8} Hz) and 71 d (1.63×10^{-7} Hz) as estimated by the LSSM AR[4] model. The line gives the estimated LSSM AR[4] spectrum. Both axes are scaled logarithmically in order to allow a direct identification of periodic and non-periodic components of the process. For a linearly scaled periodogram, Buchler et al. (1996) should be consulted.

REFERENCES

Buchler J. R., Kovacs G., 1987, ApJ, 320, L57
Buchler J. R., Kollath Z., Serre T., Mattei J, 1996, ApJ, 462, 489
Honerkamp J., 1993, Stochastic Dynamical Systems. VCH Publ., New York
König M., 1997, PhD thesis, University of Tuebingen, Germany
König M., Timmer J., 1997, A&AS, 124, 589
König M., Staubert R., Wilms J., 1998, A&A, 326, L25
Kukarkin B. V., 1975, Pulsating Stars. Wiley, New York
Priestley M. B., 1992, Spectral Analysis and Time Series. Academic Press, San Diego
Scargle J. D., 1981, ApJS, 45, 1
Yule G., 1927, Phil. Trans. R. Soc. A, 226, 267

This paper has been typeset from a TEX/LATEX file prepared by the author.