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Analyzing the dynamics of hand tremor time series

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Abstract. We investigate physiological, essential and parkinsonian hand tremor measured by the acceleration of the streched hand. Methods from the theory of dynamical systems and from stochastics are used. It turns out that the physiological tremor can be described as a linear stochastic process, and that the parkinsonian tremor is nonlinear and deterministic, even chaotic. The essential tremor adopts a middle position, it is nonlinear and stochastic.

1 Introduction

Tremor time series are usually analyzed by estimating the power spectrum. Such an analysis of the power spectrum is well established for any kind of time series if they can be considered as stationary. The analysis determines the dominant periodicities, but there are many other features that describe a time series and may hence serve as a characterization of tremor.

Within the general context of time series analysis one distinguishes between deterministic and stochastic processes. A deterministic process can be described by a mathematical equation which determines a value of a quantity by previous values. For such processes the future behavior can be predicted in principle completely. There are, however, systems for which this is impossible in practice, these deterministic processes are called chaotic. Their specific property is their sensitivity to small influences: if one considers two time series of such a process which may be identical up to a certain time and, if one changes the value of one time series by an arbitrary small amount, then in the future the two time series will differ more and more. The difference of the time series can be shown to increase locally exponentially, and time series recorded from chaotic processes show always irregular behavior. Hence, because in reality any system is measurable only with finite precision, chaotic processes are unpredictable in spite of being deterministic.

Besides the deterministic approach in describing phenomena in nature there is the stochastic ansatz. This is motivated by the idea, that most systems in nature, which are complex enough, are always exposed to so many uncontrollable influences that a mathematical equation for the dynamics of the system ought to contain random quantities. Random variables in mathematical context are not completely arbitrary but are characterized by a distribution, so that the characteristic features of the random variables determine the kind of the external influence. Because of the random character each realization, i.e. recorded time series, of a stochastic process looks different.

Given a time series that shows irregular behavior as e.g. any tremor time series, it is not possible to decide by visual inspection whether the underlying process should be described by a deterministic or a stochastic equation. Such a decision can only be made after a mathematical analysis of the data. An application of such a test to tremor time series is the first topic of this paper.

There is another distinction that can be made for processes, either deterministic or stochastic ones. This distinction concerns the nature of the mathematical equation and therefore is a bit more abstract. The value of the measured quantity may depend *linearly* or *nonlinearly* on previous values (and on some random numbers if the process is stochastic). This form of dependence affects the appearance of the time series and therefore it should be somehow detectable. This is the second topic of this paper. We applied a test of linearity to tremor time series.

These two investigations, the analysis on stochasticity, and the test on linearity do not only provide a characterization of the time series, the finding that a time series can be considered as a realization of a linear stochastic process will open the possibility to a much more detailed description of the dynamics of the process. The theory of linear stochastic processes is well established as well as the methods to identify the process from a given time series.

It will turn out that the time series from persons with physiological tremor can be recognized as realizations of a linear stochastic process and therfore these time series can be analyzed in detail.

The time series of parkinson patients on the other side are identified as signals of chaotic systems, which are nonlinear and deterministic. Because methods for analyzing the dynamics of such nonlinear systems are not yet developed we must be satisfied with calculating the correlation dimension and the Lyapunov exponents of such signals.

Finally the essential tremor will adopt a position in the middle between the two extremes, the physiological and the parkinsonian tremor. It corresponds to a nonlinear stochastic system, which up to now can not be analyzed further.

The paper is organized as follows. In Sect. 2 the data material is described. Section 3 is devoted to a brief description of the methods, without going too much into mathematical detail. In Sect. 4 the results are reported.

2 The data

The time series are recordings of the acceleration of a hand. The sampling rate is 300 Hz. The hand is stretched out and either unloaded or loaded with weights of 500 g or 1000 g. The length of the recording is about 35 s so that 10240 data points were obtained.

The measurements were obtained from persons, who exhibit physiological tremor (normal persons) and persons with clinically diagnosed essential or parkinsonian tremor. The time series were recorded at the Neurologische Universitätsklinik of Freiburg and the data were kindly made available to us by Professors Lücking and Deuschl. Examples of the time series to be analyzed are shown in Fig. 1a–c.

The data are contaminated with noise produced by the electronics. Because the variance of the noise is constant, the signal-to-noise ratio depends on the variance of signal. In case of the physiological tremor, which shows the lowest variance among the kinds of tremor, the noise contributes up to 30% of the variance of the observed time series.

3 The methods

In this section we will briefly report the three methods we have applied to the tremor time series. The first method, the calculation of the correlation dimension, which provides a test on stochasticity, originates from the theory of chaotic systems, whereas the other two belong to the field of stochastics and are used in the time series analysis of stochastic signals.

3.1 Correlation dimension

By calculating the correlation dimension one is enabled to decide whether a time series is generated by a dynamics with a few or with many degrees of freedom. In the first case the dynamics is called low dimensional

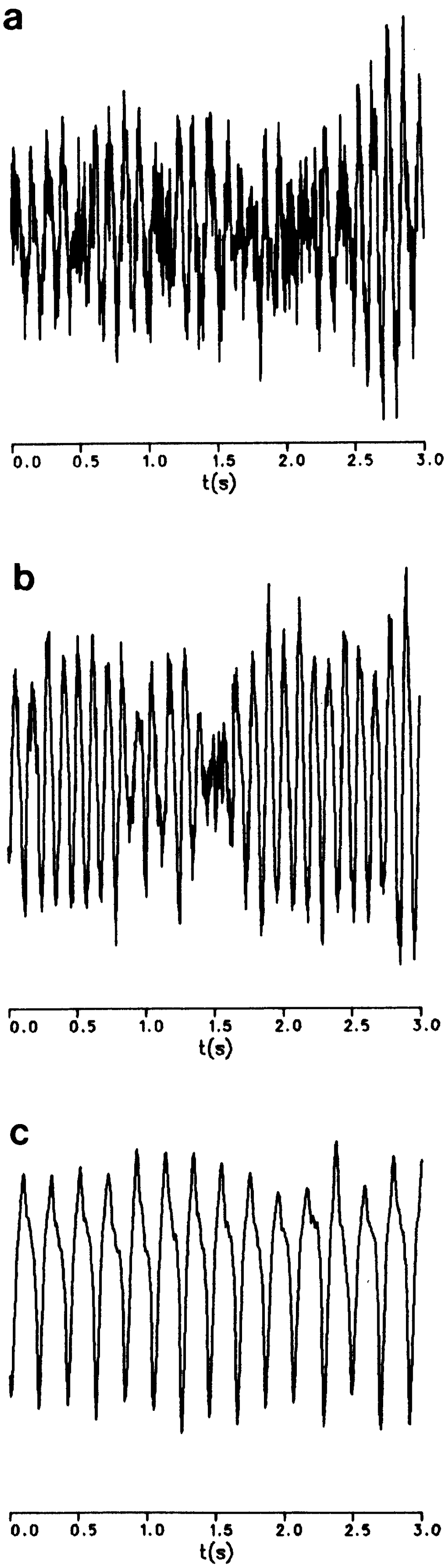

Fig. 1. Acceleration of the hand in case of physiological tremor (**a**), essential tremor (**b**) and parkinsonian tremor (**c**)

and the time series originates from a deterministic system. It is then possible to distinguish between regular (i.e. periodic) and chaotic dynamic, because a regular dynamics results in an integer correlation dimension, whereas a chaotic dynamics results in a noninteger correlation dimension. In case of many degrees of freedom it is adequate to model the time series by a stochastic process, for stochastic processes have an infinite number of degrees of freedom.

The correlation dimension provides an upper bound for the number of variables to describe the dynamics of the time series. For systems with a dimension of n there exists a representation by $2n$ variables.

In order to calculate the correlation dimension introduced by Grassberger and Procaccia (Grassberger and Procaccia 1983a, b), it is necessary to reconstruct the trajectory in phase space from the one dimensional time series $(x_t, t = 1, \ldots, N)$. Following Takens (1981) this reconstruction is obtained by forming the vector $\vec{\mathbf{x}}(t)$:

$$\mathbf{x}(t) := (x_t, x_{t+\tau}, \ldots, x_{t+(n-1).\tau}), \tag{1}$$

with an appropriate delay time τ.

The correlation dimension D is defined by:

$$D := \lim_{r \to 0} \frac{\ln(C(r))}{\ln(r)} \tag{2}$$

with

$$C(r) := \lim_{n \to \infty} \frac{1}{N^2} \sum_{i,j=1}^{N} \Theta(r - |\mathbf{x}(i) - \mathbf{x}(j)|). \tag{3}$$

The resulting dimension does not depend on the norm $|.|$ chosen. For computational reasons we choose the maximum norm. $C(r)$ is called correlation integral and is estimated by:

$$\hat{C}_n(r) := \frac{1}{N^2} \sum_{\substack{i,j=1 \\ i \neq j}}^{N} \Theta(r - \max_{k=0,\ldots,n-1} |x_{i+k\tau} - x_{j+k\tau}|). \tag{4}$$

The dimension is calculated in a range of r, where the following scaling law holds:

$$\ln(\hat{C}_n(r)) = D \cdot \ln(r) + \text{const}. \tag{5}$$

There are two main limitations for this procedure. One consists in the finite number of available data points, which determines an upper bound of the obtainable correlation dimension (Ruelle 1990) and gives a restriction to the accuracy of the estimated value (Nerenberg and Essex 1990). The other is given by the noise, that causes the correlation integrals to behave for small values of r as if they were estimated from a pure stochastic time series. If a deterministic process is contaminated by such an amount of noise, that no range of r exists, where (5) holds, no finite correlation dimension can be obtained and the system is also considered as stochastic.

An additional feature to describe deterministic processes is given by the Lyapunov exponents. They quantify the mean local divergence of nearby trajectories. In case of regular systems they all are less or equal to zero. For chaotic systems at least one of them is positive. For calculating the exponents we used the algorithm developed by Eckmann and Ruelle (1985).

3.2 Test for linearity

As a linear dynamics we consider here the linear state space model. This class of models includes the so called ARMA-processes, but also enables us to model an additive gaussian observational noise, as produced by measurement electronics.

The wellknown power spectrum $f(\omega)$ is given by the Fourier transform of the autocorrelation function $c(t)$,

$$c(t) := \langle (x_\tau - \langle x \rangle) \cdot (x_{t+\tau} - \langle x \rangle) \rangle \tag{6}$$

$$f(\omega) := \frac{1}{2\pi} \cdot \sum_{t=-\infty}^{\infty} c(t) \cdot \exp(-i(\omega \cdot t)). \tag{7}$$

Here $\langle x \rangle$ denotes the expectation of the random variable x. Since the spectrum is only related to second moments, it is not sensitive to a possible nonlinearity of the dynamics. As an extension of the power spectrum the bispectrum $f(\omega_1, \omega_2)$ is defined by the Fourier transform of the triple correlations $c(t_1, t_2)$:

$$c(t_1, t_2) := \langle (x_t - \langle x \rangle) \cdot (x_{t+t_1} - \langle x \rangle) \cdot (x_{t+t_2} - \langle x \rangle) \rangle \tag{8}$$

$$f(\omega_1, \omega_2) := \frac{1}{2\pi} \cdot \sum_{t_1=-\infty}^{\infty} \sum_{t_2=-\infty}^{\infty} c(t_1, t_2) \cdot \exp(-i(\omega_1 \cdot t_1 + \omega_2 . t_2)) \tag{9}$$

In case of a process with a linear dynamics which is driven by gaussian noise it can be shown that the triple correlations and hence the bispectrum is zero. Subba Rao and Gabr (1980) developed a statistical test that decides whether the estimated bispectrum is consistent with the hypothesis of the bispectrum being zero.

The main assumption for the derivation of the statistic in case of the null hypothesis of linearity is that the variance of the bispectrum depends only weakly on the frequencies. To justify the application of the test we tested the statistics of the zero hypothesis by Monte-Carlo simulations. The data for these simulations were obtained from models that show the same power spectrum and therefore exhibit the same variance of the bispectrum (Brillinger 1981) as the time series of the phsyiological tremor.

Another assumption is that the driving noise of the process is gaussian. If the time scale of the physical process, that produces the noise, is small compared with the sampling time, the noise is averaged and therefore it becomes gaussian distributed on the time scale of the sampling. Since we assume the signal of the motor neurons to be the source of the noise this assumption is satisfied.

Thus this test provides a reliable method to detect deviations from linearity.

3.3 *Time series analysis by state space models*

The linear state space model consists of a hidden multivariate AR[1]-process $\mathbf{x}_t$ that is observed via a matrix C. The observation y_t is disturbed by an additive noise η_t. Hence the equations of dynamics read:

$$\mathbf{x}_t = A\mathbf{x}_{t-1} + \boldsymbol{\epsilon}_t$$
$$y_t = C\mathbf{x}_t + \eta_t \tag{10}$$

We assume that the random variables representing the noise are normally distributed with mean zero and variances:

$$\langle \boldsymbol{\epsilon}_t \boldsymbol{\epsilon}_{t'}^T \rangle = Q \cdot \delta_{t,t'}$$
$$\langle \eta_t \eta_{t'} \rangle = R \cdot \delta_{t,t'}$$

This model enables us to treat the additive noise of the electronics explicitly by η_t. In case of fitting a pure AR-process such an additive noise would cause artefacts. The dynamics of the system are given by the multivariate AR[1]-process that can be interpreted as a system of linear damped oscillators and linear relaxators driven by the noise $\boldsymbol{\epsilon}_t$. The frequencies ν and relaxation times τ of this oscillators resp. relaxators are given by the eigenvalues λ_i of the matrix A:

$$\tau_i = -2/\ln(|\lambda_i|^2) \tag{11}$$

$$\nu_i = \arctan\left(\frac{\Im(\lambda_i)}{\Re(\lambda_i)}\right), \tag{12}$$

where $\Im(\lambda)$ denotes the imaginary part and $\Re(\lambda)$ the real part of the complex eigenvalue λ.

Since the noise is gaussian and the model is linear the likelihood can be formulated explicitly. This expression depends nonlinearly on the parameters A, Q, C, R so that the maximum likelihood estimators have to be calculated numerically. For reasons of numerical stability we used an EM-algorithm (Shumway and Stoffer 1983).

To test the adequacy of the fitted model we analyze the errors of one-step predictions. The time series of this errors is tested for white noise by the Komogorov-Smirnov test (Brockwell and Davis 1987).

The obtained models were used to produce the data in the above described Monte-Carlo simulations referring to the bispectrum.

4 The results

4.1 *Physiological tremor*

The calculation of the correlation dimension in case of physiological tremor shows that the dynamics are stochastic.

The test of the bispectrum has been restricted to a range of frequencies between 3 Hz and 9 Hz. In this range almost all of the variance of the process is located (applying the test up to higher frequencies would only test the noise of the electronics). The bispectrum is assumed to be sufficiently constant within a range of 0.5 Hz. Taking these parameter Monte-Carlo simulations show no deviation from the theoretically calculated statistics.

The test establishes the result that the bispectrum of this class of time series is consistent with zero. This justifies the hypothesis that the dynamics are linear and thus also excludes the possibility of the dynamics being chaotic. Hence we try to model the time series by a linear state space model according to (10).

In most cases a state space model with a two dimensional hidden AR[1]-process is successful, judged by the whiteness of the prediction errors. By (11) and (12) the estimated parameters can be interpreted as a linear damped oscillator driven by noise. This mathematical model can be translated into medical terms: the mechanical system of the hand forms a damped oscillator and the noise is equivalent to the uncorrelated activated motoneurons. The power spectra of the time series are in good agreement with the power spectra calculated analytically from the parameters of the models (Fig. 2).

If the hand is loaded with weights of 500 g resp. 1000 g, the period of the resulting physiological tremor is proportional to the square root of the loaded weights (Elble 1986). This relationship is reproduced by our fitted models with periods calculated following (12) (Fig. 3).

In some cases a four dimensional model corresponding to a system of two oscillators was necessary to describe the data adequately. In contrast to the above mentioned time series the spectra of these data show a different behavior under increasing the load weights. While the former consists in one peak moving under loading, the latter exhibit a splitting of the peak into one small stationary peak and one moving as in the former case. Because of this behavior the moving

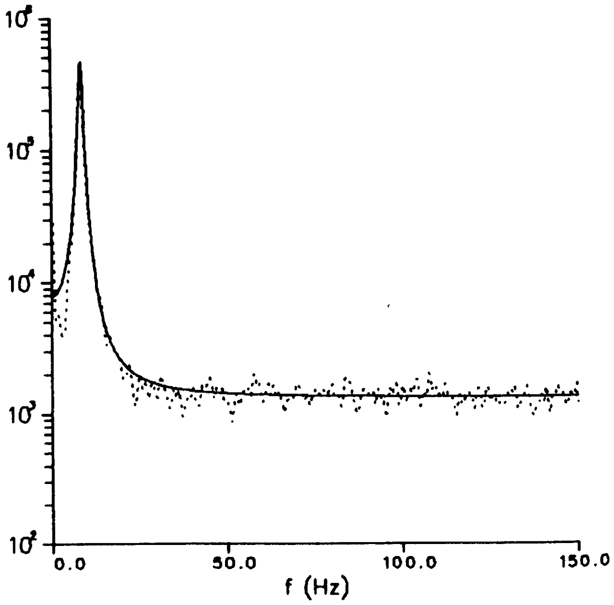

Fig. 2. Power spectrum in case of physiological tremor (*dotted*), power spectrum of the corresponding state space model (*solid*)

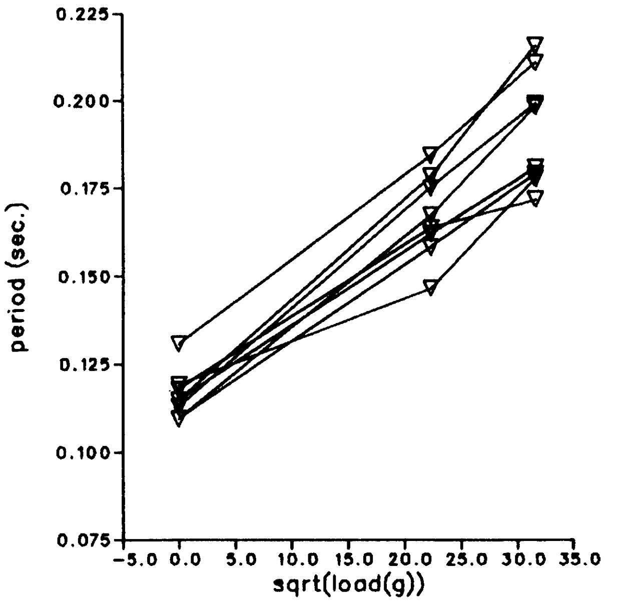

Fig. 3. Periodicities calculated from state space models fitted to physiological tremor in dependence on the weight loaded on the hand

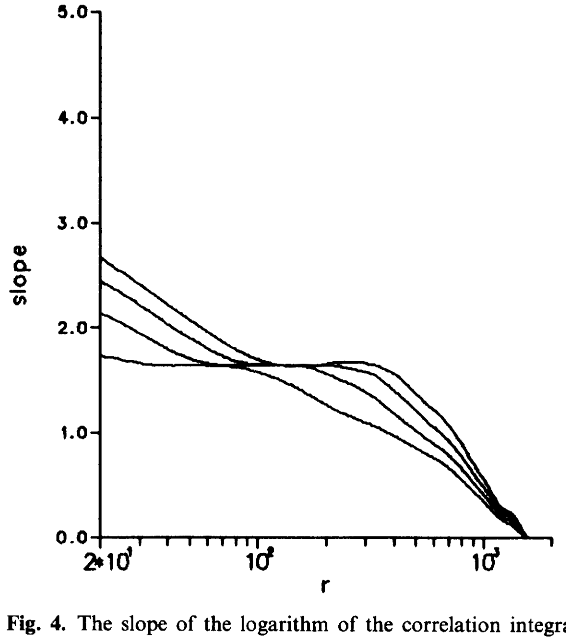

Fig. 4. The slope of the logarithm of the correlation integrals $\hat{C}_n(r)$ vs. $\ln(r)$ for data from a parkinsonian tremor. Curves are shown for $n = 2, 3, 4, 5$. If the slope exhibits a plateau the value of the slope gives the correlation dimension. In this case we obtain a value of 1.6. The raise of the slope for small values of r is caused by noise

peak is called motoric peak and the stationary peak is called synchronisation peak.

4.2 Parkinsonian tremor

The correlation dimension of parkinsonian tremor is proven to be finite and noninteger so that the dynamics is regarded as deterministic chaos. The calculation of the Lyapunov exponents supports this result because one of them is always positive.

Five of eight time series under consideration show a low correlation dimension from minimum 1.4 ± 0.1 to maximum 1.6 ± 0.1. The delay time τ in (1) was chosen to be 1/150 s. In these cases a large enough range was found where the relation (5) holds. For small values of r the behavior of the correlation integral is dominated by the noise (Fig. 4). For the remaining three time series the noise was too strong to obtain a finite correlation dimension.

The calculation of the Lyapunov exponents is a bit critical for technical reasons. The obtained values depend sensitively on the parameters for the reconstruction of the trajectory (1). In spite of these difficulties one comes to the result that for each of the above mentioned five time series with reasonable chosen parameters always one Lyapunov exponent is positive (Fig. 5).

4.3 Essential tremor

The power spectrum of the essential tremor exhibits not only the motoric peak known from physiological tremor but also a strong synchronisation peak (Elble 1986). The motoric peak in case of physiological tremor is recognised as caused by a linear stochastic process. Therefore the essential tremor is expected to include a strong stochastic component.

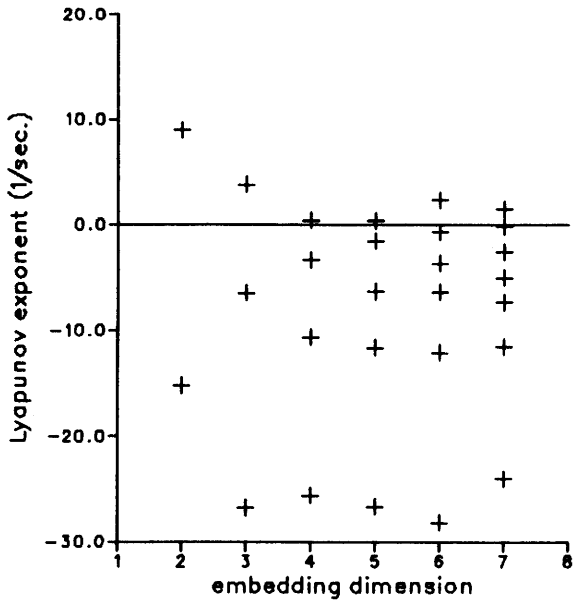

Fig. 5. The Lyapunov exponents calculated from a time series of a parkinsonian tremor for different embedding dimension. The delay time τ is $4\Delta t$ corresponding to 1/75 s

Because of this stochastic contribution of the motoric peak it was not possible to decide whether the synchronisation peak is governed by a deterministic dynamics or not. A typical result of estimating the correlation integral is shown in Fig. 6.

In contrast to the physiological tremor the test of the bispectrum in most cases rejects the hypothesis of

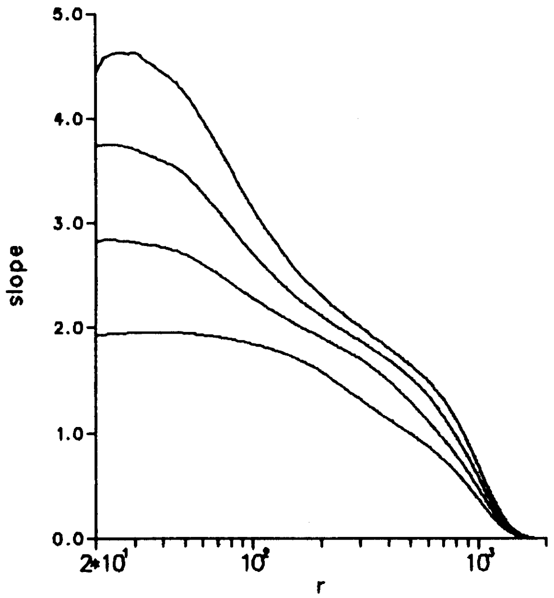

Fig. 6. The slope of the logarithm of the correlation integrals $\hat{C}_n(r)$ vs. $\ln(r)$ for data from essential tremor with $n = 2, 3, 4, 5$. The slope exhibits no plateau. Therefore no finite correlation dimension can be obtained

zero bispectrum. From this we may infer that the data are caused by a nonlinear dynamics.

5 Summary

We applied methods from stochastics and from the theory of dynamical systems to investigate physiological, essential and parkinsonian hand tremor, measured by the acceleration of the stretched hand.

By fitting linear state space models the physiological tremor could be identified as a linear damped oscillator driven by white noise. In some cases two such oscillators were neccessary to describe the data sufficiently. The linearity of the dynamics was proven by a statistical test on the bispectrum.

In contrast to the stochastic and linear dynamics of the physiological tremor the parkinsonian tremor was recognized as a deterministic system with a nonlinear dynamics. This result was obtained by a calculation of the correlation dimension that turned out to be finite and noninteger. The estimation of the Lyapunov exponents supports this result.

In the case of essential tremor a finite correlation dimension was not obtainable. The test of the bispectrum suggests the postulation of a nonlinear dynamics.

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Artificial neural network classification of *Drosophila* courtship song mutants

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Abstract. Courtship songs produced by *Drosophila* males – wild-type, plus the *cacophony* and *dissonance* behavioral mutants – were examined with the aid of newly developed strategies for adaptive acoustic analysis and classification. This system used several techniques involving artificial neural networks (a.k.a. parallel distributed processing), including learned vector quantization of signals and non-linear adaption (back-propagation) of data analysis. "Pulse" song from several individual wild-type and mutant males were first vector-quantized according to their frequency spectra. The accumulated quantized data of this kind, for a given song, were then used to "teach" or adapt a multiple-layered feedforward artificial neural network, which classified that song according to its original genotype. Results are presented on the performance of the final adapted system when faced with novel test data and on acoustic features the system decides upon for predicting the song-mutant genotype in question. The potential applications and extensions of this new system are discussed, including how it could be used to screen for courtship mutants, search novel behavior patterns or cause-and-effect relationships associated with reproduction, compress these kinds of data for digital storage, and analyze *Drosophila* behavior beyond the case of courtship song.

1 Introduction

With our growing understanding of the cellular basis of behavior and the advent of newer, more powerful techniques for neurophysiological and molecular analyses, animal behavior is being increasingly elucidated at the cellular and subcellular levels. However, a major difficulty in the investigation of behavior is the requirement to integrate, for a given analysis, a large number of observable variables (internal as well as external) whose relation to one another is not necessarily understood. Conventional multivariate statistical approaches (e.g., Bayesian prediction) suffer from their dependence on large sample numbers and from their limitations on non-normally distributed data whose variables may exhibit non-linearities. This constraint precludes the use of general analytic techniques for the study of complex behaviors.

On the other hand, artificial neural networks (ANN) have successfully been used for studying such kinds of multivariate problems. Applications in which ANN techniques have proven useful include sonar classification (Gorman and Sejnowski 1988), speech synthesis (Rosenberg and Sejnowski 1987), machine-based visual recognition (Fukushima et al. 1983; Menon and Heinemann 1988), control systems (Miyamoto et al. 1988), optimization and decision-making (Hopfield and Tank 1985), medical diagnoses (Bounds et al. 1990), RNA splice-site recognition (Brunak et al. 1990), climate prediction (Hu 1964; Rogers 1989), jet engine performance (Dietz et al. 1989), and economic phenomena (Werbos 1984, 1988). The irony is that although ANN are inspired and partially derived from biological nervous systems, they have not yet been extensively applied to the study of animal behavior. We describe in this paper a self-learning system which we have developed to analyze and classify the courtship songs of *Drosophila* males.

Courtship is one of the best studied *Drosophila* behaviors (reviews: Spieth 1974; Ewing 1983; Tompkins 1984); and because it has a tractable number of behavior components (e.g. Hall 1986), it is ideal for comprehensive quantitative and comparative analyses (e.g., Cook 1979, 1980; Markow and Hanson 1981). Several of these courtship elements define differences among various species of *Drosophila* (e.g., Cowling and Burnet 1981; Wheeler et al. 1988; Hoy et al. 1988).

We have been investigating the genetic and neural mechanisms underlying courtship as well as the adaptive significance of this behavior (e.g. Hall 1979; Schilcher and Hall 1979; Greenspan et al. 1980; Siegel

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